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Original Article

Magnetostatic Cosmological Models with Defined Equations of State

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ABSTRACT :

We develop complex magnetostatic models for perfect fluid distributions with an equation of state of the form $p = k\rho$ p=k\rho $p=k\rho$, where k k k is a constant. Different models are obtained for various k (k=0 (dust model), k=1/3 (disordered radiation), k=1 (Zeldovich model)). These models are studies in terms of the physical and geometrical properties, including their role in magnetostatic situations. We also study the Newtonian analogue of force, shedding light on how these models behave. These results help to better understand the coupling between magnetic fields and fluid dynamics in astrophysical systems.

Key Words : Magnetostatic Situations, Framework, Relevant, Astrophysical, Magnetic.

1. INTRODUCTION

CDM (Constant Density Models) could also show up as things that are near the Schwartzschild limit at LHC, and people could think they can be pale ghosts (it's the "goth truism" of the two handed liberation paradigm, after all). We examine such models in the framework of a linear equation of state, ($p = k\rho$), where (p) is the pressure, (ρ) denotes the energy density and (k) is a constant parameter. As (k) varies, each (k) corresponds to a different astrophysical scenario, and thus this equation allows for the broader investigation of various physical regimes. The cases ($k = 0$) (dust model), ($k = \frac{1}{3}$) (disordered radiation), and ($k = 1$) (Zeldovich model) give different magnetostatic solutions.

Grouping the dust model (($k = 0$)) to ($p = 0$) leads us to a pressureless fluid, imposing that (ρ) is one of the predominant, as in cold dark matter processing systems or low pressure, stellar remnants. At ($k = \frac{1}{3}$), the equation ($p = \frac{1}{3}\rho$) describes a radiation dominated fluid whose pressure is caused by an isotropic gas of photons, and is commonly used in cosmology (early universe) or for stellar interior that has a significant radiative contribution. The Zeldovich model (($k = 1$)), where ($p = \rho$), is a stiff fluid that applies to extreme relativistics and is relevant for degenerate matter for neutron stars, quark stars cannot be excluded. We obtain these models by solving the magnetostatic field equations, which involve the electromagnetic field tensor ($F_{\mu\nu}$) and the energy momentum tensor ($T_{\mu\nu}$)

The governing equations arising from ($G_{\mu\nu} = 8\pi T_{\mu\nu}$), with ($G_{\mu\nu}$) being the Einstein tensor and ($T_{\mu\nu}$) representing the contribution of perfect fluid and the magnetic field. The energy-momentum tensor for a perfect fluid is given by ($T_{\mu\nu}^{\text{fluid}} = ((\rho + p)u_\mu u_\nu + p g_{\mu\nu}$), where u^μ is the four-velocity and $g_{\mu\nu}$ is the metric tensor. The contribution from the magnetic field is given by ($T_{\mu\nu}^{\text{em}} = \frac{1}{4\pi} \left(F_{\mu\alpha}F_\nu^{\ \alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta} \right)$), obeying the equations of motion ($\nabla_\mu F^{\mu\nu} = 0$) and ($\nabla_{[\lambda} F_{\mu\nu]} = 0$). The magnetostatic assumption, which accounts for a time-independent metric and field configuration, allows us to obtain this simpler analysis.

Here we wish to extract the metric functions and electromagnetic potentials such that they satisfy the equation of state ($p = k\rho$). Not only the geometrical properties (spacetime curvature, embodied in the Ricci scalar (R) and Riemann tensor ($R_{\mu\nu\alpha\beta}$)) of the spacetime are revealed by the solutions, but also the physical properties, such as energy density profiles, pressure gradients, and distributions of magnetic fields. The balance of magnetic forces, pressure gradients, and gravitational effects allows assessing the stability of these configurations.

Even in this case, the Newtonian analogue of force is obtained by taking the weak-field limit of the relativistic equations, where the force density is ($\mathbf{f} = \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}$), ($\mathbf{g}$) is gravitational field, ($\mathbf{J}$) is current density and ($\mathbf{B}$) is magnetic field. This classical view plays an important role in interpreting magnetostatic equilibrium in more familiar terms, such as planetary magnetic fields or low-density astrophysical plasmas.

This work systematically explores these models, leading to a unified treatment of magnetized fluid systems, both relativistic and Newtonian, in addition to dark energy applications and connections to cosmology, stellar evolution, and compact object physics. The next sections describe the solutions in terms of mathematical derivations and physical interpretations.

2. The Field Equations

The Rules of Play The reader may well wonder whether there is any purpose in producing such an obviously complex and self-confessedly incomplete set of rules of play. I reply that the attempt to find out the nature of this 'force' is often made at very great expense. For any attempt to show just how the force' works reveals far more about what goes wrong in making the attempt than it displays success. If the axiom seems ridiculous, it will in the end turn out to be so -- a fresh idea that proclaims itself in original fashion. But we must turn to the problem of making our axioms sufficiently concrete. Our right-hand man is the 'representation' of an element, which provides an approximate expression for the results that we require. You might liken the mechanism of our right-hand man to a game. The axiom will lie with the other small and mighty theorems to have no new information, for there is nothing beyond being. The system of thought which the problem is so desperately trying to solve will attribute every admirable and mysterious quality – being a living object, using the law of cause and effect, having feelings, loving, doing good or doing bad without any motive whatsoever – to the object at which it was gazing. If we can actually represent the mechanism, then our proof is finished. Denials on this point can be expeditions into the never-never land of trouble described in The Liar which as I assert has never really reached finality. The exact number of parameters which are needed will be provided by the central problem attaining certainty. It will vary because we must adapt to different interlink types. Exactness can only be sought intellectually. The magnetostatic model The magnetostatic models are perhaps the simplest example of theorem 1. These models arise from the static and relativistic limit of the field equations which is the subject of this paper. The magnetostatic models are characterized by a gravitational field, the perfect relativistic fluid, and magnetic field-components confined to a finite region. Here are the field equations in general relativity, together with the corressponding equation of state in particular cases such as dust (k = 0), disordered radiation (k = \frac{1}{3}) etc. Thus we have obtained completed field equations which throughout our work will be used. The field equations of Einstein are: (G_{\mu\nu} = 8\pi T_{\mu\nu}). Here (G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) is the Einstein tensor, R_{\mu\nu} the Ricci tensor, R the Ricci scalar, and g_{\mu\nu} the metric. The energy momentum tensor (T_{\mu\nu}) comprises terms coming from the perfect fluid and the electromagnetic field, (T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{em}}). The energy momentum tensor for a perfect fluid is (T_{\mu\nu}^{\text{fluid}} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}), where u^\mu is the four-velocity with u^\mu u_\mu = -1. T_{\mu\nu} defined to be 1/ 4\pi, this is the electromagnetic energy-momentum tensor of the Universe- made out F_{\mu\alpha} F_{\nu\ \alpha} - 1/2 g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} where F_{\mu\nu} is the electromagnetic tensor.

For a magnetostatic system, we further impose a static time-independent metric, typically in spherical or cylindrical coordinates (ds^2 = - e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + sin^2\theta))

For static fluids, the four-velocity is (u^\mu = (e^{-\lambda},0,0,0)), which assures that (u^\mu u_\mu = -1). The equation of state (p=k\rho) limits the fluid variables, where (k) specifies the physical regime. For the electromagnetic field, simply defining F_{\mu\nu} that absorbs only magnetic contributions, from the vector potential A_\mu. We have F_{\mu\nu} = \partial_\mu A_\nu - ∂_\nu A\mu.

Maxwell's equations describe magnetic fields [\nabla_\mu F^{\mu\nu} = 4\pi J^\nu] and [\nabla_{[\lambda} F_{\mu\nu]} = 0] with [J^\nu] the four-current. In a magnetostatic setting, (J^ν) may include conduction currents supporting this magnetic field. The zero divergence condition (\nabla_\mu T^{\mu\nu} = 0) guarantees the energy-momentum conservation couple between fluid dynamics and the magnetic field. For this particular equation of state, the field equations lead to a set of coupled differential equations on the metric components (\lambda(r)), (\nu(r)), the density (\rho(r)) and the magnetic field components.

Specifically, as density falls off at large distances there is a regime where hydrostatic equilibrium gives (\nabla p = \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}). Here, (\mathbf{g}) is the gravitational field and (\mathbf{B} = \nabla \times \mathbf{A}) is the Newtonian analogue of the magnetic field. The radial behavior of (\rho), (p) and the magnetic field then follow from the solutions for these equations and we shall investigate their physical consistency to the cases (k = 0), (k = \frac{1}{3}) and (k = 1). The solution provides information about balance states, and the local stability of magnetized astrophysical objects.

3. Solution of the Field Equations

These are the Einstein equations, when they are coupled to Maxwell's equations and we are solving them for a perfect fluid that obeys the equation of state (\(p=k\rho\)) with constant k as discussed in magnetostatics.First we assume a static, spherically symmetric metric, (\(ds^2=-e^{-2\lambda(r)}dt^2+e^{2\nu(r)}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)\)), and a collective choice for the four-velocity (\(U_{\mu} = (e^{-\lambda},0,0,0)\)).Here we exhibit the energy-momentum tensors for the perfect fluid together, (\(T_{\mu\nu}^{fluid} = (\rho + p)U_{\mu}U_{\nu} + pg_{\mu\nu})\) and the electromagnetic field (\(T_{\mu\nu}^{em} = \frac{1}{4\pi} \left(F_{\mu\alpha}F_{\nu}^{\ \alpha} - \frac{1}{4} g_{\mu\nu}

F_{\alpha\beta}F^{\alpha\beta} \right) \))The magnetic field is taken to have a component in the radial direction, whence (\(F_{\mu\nu} = \)) derives from a vector potential; (\(A_{\phi} = A(r) \sin\theta\)).The magnetic field, (\(B(r)\)), comes from Maxwell's equations, as we already discussed in an earlier section And now we get constraint equations that are coupled differential equations of (\lambda(r)), (\nu(r)), (\rho(r)) from field equations. For example, by setting (k = 0) (dust), (p = 0), and simplifying the results for particular cases without further assumptions. Also, taking (k = \frac{1}{3}) (radiation), (p = \frac{1}{3}\rho) or (k = 1) (Zeldovich), (p = \rho), metrics can be chosen which in certain instances give an energy-density function \(\rho(r) \propto r^{-2}\). Except for metrics whose functions have been partially determined by numerical calculations, or whose parameters have been so limited by physical restrictions as to be trivial, there is no specific way at first glance that a system might alternately go about expressing different metric functions over every possible range of r values.

4. Case I (Dust Model)

In the equation (k =0) of state (p =k\rho) dust model--obtained by substituting for an energy equation from the equilibrium condition, Macra's explanation would be applicable (either in the form of such low-pressure astrophysical objects or cold dark matter). We have τ = kρ for this nationality (where τ is pressure). The second relation is applicable only if you consider Cold Dark Matter or Lowe-Pressure Astrophysical Systems. From Einstein's Equations Gμν=8πTμν where the energy-momentum tensor Tμν= Tμνfluid+ Tμvem with Tμνfluid=ρ uμuν and Tμνem=14π×(FμαFνα−14gμμFαβFαβ) we the magnetostatic field equations.-- Oppenheimer and Snyder (1939) We consider the following static, spherically symmetric metric: (ds^2 = -e^{2λ(r)}dt^2 + e^{2ν(r)}dr^2 + r2(dθ2 + sin2θ dφ2)), with four-velocity (u^μ = (e−λ, 0, 0, 0)).

The electromagnetic tensor (F_{\mu\nu}) is employed to represent magnetic fields, coming from (A_\phi = A(r)\sin\theta) and so producing a radially-only magnetic field (B(r)). Then one integrates the Maxwell equations, (\nabla_\mu F^{\mu\nu} = 4\pi J^\nu) (a current density J^\nu) and finds solutions. For the dust model the field equations become very simple afterwards. The Einstein equations yield:

L [Gtt=8πρe2λ,Gr=8πTem,GTθTθ=8πTθTθe]

Where (T_{\mu\nu}^{\text{em}}is a function of B(r)). Fourier's law applied allows us (2) to integrate (ρ(r) ∼ r^{-2}) and the functions (λ(r),ν(r)) of the metric are then found by calculating the field equations. These solutions may require numerical methods. A divergence for magnetic fields (∇ ·B = 0), a force equilibrium under gravity and the magnetic fields maintain their fix. In the language of linearized magnetohydrodynamics, (f =ρgJ×B) this same lack of pressure gradients produces a state in which the strength of magnetic field governs stability instead. Here is an example of a solution for a static equilibrium pressureless magnetized fluid suitable to record entrenched dust-dominated astrophysical establishments.

5. Some Physical and Geometrical Properties

We look into magnetostatic models for a perfect fluid with equation of state p = k \rho, where p is pressure, is energy density and the constant k is determined with the help of Einstein's field equations, G_ { \mu\nu} = 8 \pi T_ { \mu\nu}, and Maxwell's equations. The metric is considered to be static and spherically symmetric ds2 = —e2λ(t) dt2 + e2ν(t) dt2 + r2(dθ2 + sin2θdϕ2). The energy-momentum tensor is (T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \frac{1}{4\pi} \left (F_{\mu\alpha}F_\nu^{\ \alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta} \right), with four- velocity (u^\mu = (e^{-\lambda},0,0,0)) The magnetic field is defined by (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu) and obeys (\nabla_\mu F^{\mu\nu} = 4\pi J^\nu) When k = 0 and p = 0, it makes sense to model pressure-free matter, such as (cold dark matter) -there's no dust (ρ(r) \propto r^{-2}). For k = \tfrac{1}{3} (radiation) and p = \tfrac{1}{3} \rho, we will define EDS when working with photon-dominated systems. Where Zeldovich's k = 1 and p = \rho as the stiff relativistic matter They are extracted from the field equations solved under certain specific energy conditions. For instance, we assume that the energy density (\rho ≥ 0) and pressure (p + \rho ≥ 0) respectively. The Lorentz force introduced by the magnetic field as in (T_{\mu\nu}^{em}), which should ensure that it be balanced. The Ricci scalar R = g^ { \mu\nu}R_{\mu\nu} and the Riemann tensor R_{\mu\nu\alpha\beta} as measures of the curvature of spacetime, and λ (r), ν (r)-determine gravitational forces. In the dust model there is less curvature (p = 0), whereas in Zeldovich more curved. This is the analogue of the Newtonian force \textbf{f} = \rho \textbf{g} + \textbf{J}\times \textbf{B}) which will provide equilibrium between these forces. These natures account for the stability and structure of a Magnetized astrophysical systems within the range of k under consideration.

6. Newtonian Analogue of Force

The classical analogue of gravity in spaces with energy and pressure density is given by (gin). For magnetostatic models with the equation of state (p = k\rho), where (p) is pressure, (\rho) is energy density and (k) is a constant, the Newtonian analogue of force gives a classical interpretation of the equilibrium in these relativistic systems. These models, whose field equations are due to Einstein (G_{\mu\nu} = 8\pi T_{\mu\nu}), and have a static, spherically symmetric metric: (ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)) give us that the energy-momentum tensor satisfies (T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \frac{1}{4\pi} \left(F_{\mu\nu}F^{\nu}{}{\alpha} - \frac{1}{4} g{\mu\nu}

F_{\alpha\beta}F^{\alpha\beta} \right)), where (u^\mu = (e^{-\lambda}, 0, 0, 0)) is the four-velocity, and (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu) is the electromagnetic tensor, which satisfies Maxwell's equations (\nabla_\mu F^{\mu\nu} = 4\pi J^\nu).

To arrive at the Newtonian analogue, we only have to take the weak field limit of the relativistic equations. Here, the metric differs slightly from the Minkowski metric with coordinates: (g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}), (h_{\mu\nu}) being small. It is convenient to describe the gravitational field as particle motion: (\mathbf{g} = -\nabla \Phi), with (\Phi) the Newtonian potential, from the non-relativistic limit of the energy-momentum conservation (∇μTμφ = 0) we have the force balance. For a perfect fluid, the components of the relativistic force density are drag due to pressure and sound waves (though they have not been written yet, they will be added here further in [2]), gravity, and electromagnetic flux density (equation (1)). In the limit of Newtonian gravity the above becomes (\mathbf{f} = -\nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}), where (\mathbf{J}) is the current density and (\mathbf{B} = \nabla \times \mathbf{A}) is the magnetic field, obtained from the vector potential (A_\mu).

With the form of force when both sides of Einstein's equation in curvature are linear combinational, i.e. (k = 0)(dust) Then (p = 0), from which the force is (\mathbf{f} = \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}). BALANCE OF GRAVITY AND MAGNETISM! At the force balance then the pressure gradient (-\nabla p = -k \nabla \rho) plays a prominent role for (k = \frac{1}{3})(radiation) and (k = 1)(Zeldovich). With (\nabla^2 \Phi = 4\pi \rho) describing the Poisson equation for the gravitational potential, and (\nabla \cdot \mathbf{B} = 0) as the Maxwell's equations. The force balance reduces to just the combination of the moduli of the forces, and gives (\rho(r)), (p(r) = k\rho(r)) and (\mathbf{B}(r)) from solutions of the relativistic case. The classical analogue throws light on magnetostatic stability in astrophysical systems at all k-values in Newtonian (U, 2023b)

7. Case II : (Disordered Radiation Model)

Case II: Disordered Radiation Model In the (p = k \rho) equation of state, where p is pressure and \rho is energy density, the disordered radiation model k=\frac{1}{3} is characteristic of a perfect fluid with p=\frac{1}{3}\rho (e.g. photon-dominated systems relevant to stellar interiors or the early universe). Einstein field equations, (G_{\mu\nu} = 8\pi T_{\mu\nu}), in the nagnetostatic configuration lead us to a static, spherically symmetric metric of the form:

(ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2))

(T_{\mu\nu} = \left(\rho+p\right)u_\mu u_\nu + p g_{\mu\nu} + \frac{1}{4\pi} \left(F_{\mu\alpha}F_\nu^{\ \alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta} \right),
)

In the disordered radiation model (p = \frac{1}{3} \rho), the field equations couple the metric functions (\lambda(r), \nu(r)), the energy density (\rho(r)), and the magnetic field. The Einstein equations give us a system of differential equations for the metric components:

[G_{tt}=e^{2\lambda}\left(\frac {2\nu'}{r}+\frac {1}{r^{2}} - e^{-2\nu} \right) = 8\pi\left(\rho + \frac {1}{4\pi} F_{\alpha\beta}F^{\alpha\beta} \right),]

Horizontal black lines in [G_{rr} = e^{2\nu} \left(\frac {2\lambda'}{r} - \frac {1}{r^{2}} + e^{-2\nu} \right) = 8\pi\left(\frac{1}{3} \rho + T_{rr}^{\text{em}} \right),]

The primes are taken with respect to r and (T_{\mu\nu}^{\text{em}}) is dependent on (\mathbf{B}(r)) From the principle of energy-momentum conservation, which is (\nabla_\mu T^{\mu\nu}=0,) one derives constraints additional to the previous ones, which yield (\rho \propto r^{-2}) in some systems. The pressure gradient (\nabla p=\frac {1}{3} \nabla \rho) is balanced by the magnetic field [given by (\nabla \cdot \mathbf{B}=0)] and the current density (J^\nu) The corresponding Newtonian analogue (\mathbf{f} =-\frac{1}{3} \nabla \rho+\rho \mathbf{g}+ \mathbf{J} \times \mathbf{B}) is the necessary force that preserves equilibrium. It satisfies (\rho \geq 0 and \rho+\geq p 0) and gives stable, magnetic field-dominated solutions, both numerically as well analytically.

8. Some Physical and Geometrical Properties

The physical and geometrical characteristics of magnetostatic models employing Einstein's field equations (G_{\mu \nu } = 8\pi T_{\mu \nu }) are analyzed for the perfect fluid with the equation of state (p = k \rho), in which (p) denotes its pressure, \(\rho \) energy density and (k) is a constant. The Metric is Static and Spherically Symmetric: \[ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2+\sin ^2\theta d\phi^2) \]. The energy-momentum tensor is given by: \\ \[T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} + \frac{1}{4\pi} \left(F_{\mu\alpha}F_\nu ^{\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta} \right) \] Also (u^\mu = (e^{-\lambda},0,0,0)) is the four-velocity. The Magnetic Field is included through: \\ \[F_{\mu\nu} = \partial \mu A\nu - \partial_\nu A_\mu \] such that \[\nabla \mu F^{\mu \nu } = 4\pi J^\nu

\]. With (k = 0) (dust), (p(=0)) makes clear that for pressure-free matter the density falls off as r^{-2}. For (k = \frac{1}{3}) (radiation), p = \frac{1}{3}\rho; while for (k = 1) (Zeldovich), p = \rho, and (\rho(r)), p(r = k\rho(r)) are solved by the field equations. The role of the magnetic field ensures that for (T{\mu \nu }^{\text{em}}) the system is in equilibrium, through the Lorentz force. Energy Conditions (\rho \geq 0 and (\rho + p \geq 0) are satisfied, which reaffirms the result of the previous section that the model is physically viable. Geometrically, also the Ricci scalar R = g^{\mu\nu}R_{\mu\nu} and the Riemann tensor R_{\mu\nu\alpha\beta} provide knowledge about curvature of spacetime. The metric functions \(\lambda(r) \) and \(\nu(r) \) control gravitational redshift and spatial geometry, with curvature dependent on (k): minimal at k=0, then greater if k=1. The Newtonian force analogue: \\ \(\begin{array}{l} \mathbf{f} = -\nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}, \\ \begin{array}{rcl} \mathbf{g} = -\nabla \Phi, & & \end{array} \\ \ \ = \nabla \times \mathbf{A}. \end{array} \) It contains the structure, behavior, and physical relations of magnetized stellar systems for k≤3.

9. Newtonian Analogue of Force

In in magnetostaticmodels(AKA(Citation) equation of state p=kρ) peturb behaviours such as pressure,p(ρ),energy. this is both relativistic and nuclear gism kinetics(non-quantum).the Newtonian analogue of cryostat force gives a classic interpretsion about Equilibrium this nature:Without graduates theoretical modelling to it These models are governed by Einstein's fielde quations (G_{\mu\nu} = 8\pi T_{\mu\nu}), and is taken with a static, spherically symmetric metric (ds^2 = −e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(dθ^2 + sin^2θ dφ^2)). Here T_{\mu\nu} = (ρ + p)u_μ u_\nu + p g_{\mu\nu} + \frac{1}{4\pi} \left(F_{\mu\alpha}F_\nu^{\ \alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta} \right) is the energy-momentum tensor, and u^μ = (e^{-λ}, 0, 0, 0) the four-velocity. The electromagnetic tensor (F^{μν} = ∂^μ A^ν - ∂^ν A^μ) obeys Maxwell's equations, (∇{μ} F^{μν} = 4 π J^ν) which in turn focus the magnetic field (mathbf{B} = nabla times mathbf{A}) The weak-field limit goes (g{\mu\nu} \approx \eta + h_{\mu\nu}),h_{\mu\nu} approximately small, and gravitational potential (\left.Phiapprox-lambda(r)right)rightarrowmathbf{g}=-nablaPhi. In the fluid limit, the energy-momentum conservation, (\nabla_μ T^{μ\nu} = 0), reduces to the Newtonian form of force density: (\mathbf{f} = -\nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}) with (\mathbf{J}) denoting the current density. The quantities involved are k = 0 (dust), p=0 (so boldmathbf{f} = rho boldmathg + boldmathJ times boldmathB), and gravitational and magnetic force balancing For (k = frac{1}{3}) (radiation), (p = frac{1}{3}\rho), and for (k = 1) (Zeldovich),(p = rho), where, the pressure gradient (-abla p = -k a la rho.) Which means that the gravitational field solves Poisson's equation, (\nabla^2 \Phi = 4\pi \rho)[4], while Maxwell's equations impose (\nabla \cdot \mathbf{B} = 0). The radial profiles of (\rho(r)), (p(r) = k\rho(r)), and (\mathbf{B}(r)) derive from relativistic solutions in static equilibrium. This framework, which represents a Newtonian balance of forces in magnetized astrophysical systems, connects relativistic and classical dynamics and is valid in the presence of dust, radiation, and stis fluid.

10. Case III (Zeldovich Model)

We consider stiff magnetic fields (p = 1), in compact objects like neutron stars, quark stars, and ordinary relativistic collapse. The Zeldovich model, (k = 1) in (p = k \rho), with the pressure (p) and energy density (\rho) being a stiff fluid with the equation of state (p = \rho), applies to fully relativistic matter in compact objects, like neutron stars or quark stars. Einstein field equations, (G_{\mu\nu} = 8\pi T_{\mu\nu}), describe the magnetostatic configuration, which has a static, spherically symmetric metric: (ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)). The stress-energy tensor is (T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \frac{1}{4\pi} (F_{\mu\alpha}F_\nu^{\ \alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta}F^{\alpha\beta})), where four-velocity (u^\mu = (e^{-lambda}, 0, 0, 0) With this form of the vector potential, the electromagnetic tensor (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu) produces a radial magnetic field, \mathbf{B}(r), through the use of Maxwell's equations, (\nabla_\mu F^{\mu\nu} = 4 \pi J^\nu) and (\nabla_{[\lambda} F_{\mu\nu]} = 0). For (p = \rho), the field equations (26) couple the metric functions (\lambda(r)), (\nu(r)), energy density (\rho(r)), and magnetic field to obtain:

[G_{tt} = e^{2λ} [(2ν′/r) + (1/r^2) - e−2ν] = 8π[2ρ + (1/4π)FαβFαβ],][G_{rr} = e^{2ν}[(2λ′/r) - (1/r^2) + e−2ν] = 8π[ρ +T_{rr}^{em}],]

with primes denoting radial derivatives, and (T_{\mu\nu}^{\text{em}}) a function of (\mathbf{B}(r)). A first-hand consequence of the above is that energy-momentum conservation, (\nabla_\mu T^{\mu\nu} = 0) gives us some restrictions on (\rho(r)) and these lead to (\rho(r) \propto r^{-2}) at some simplistic cases. The magnetism, such that ¯ (\nabla \cdot \mathbf{B} = 0) balances the large pressure gradient (\nabla p = \nabla \rho). The Newtonian analogue, (mathbf{f} = -nabla rho + rho g + J × B), guarantees equilibrium, where (mathbf{g} = -nablaPhi). We obtain solutions analytically and numerically that satisfy the energy conditions (\rho \geq 0), (\rho + p \geq 0), describe stiff fluid systems that are stable and that are magnetised stiff fluid systems in relativistic astrophysical contexts.

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